Intuitive-Calculus.com Presents

The Free Intuitive Calculus Course

Limits

Day 1: Intuitive Idea Behind Limits and Notation

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Contents

1 Welcome

Welcome to Day 1 of the Free Intuitive Calculus Course! The main purpose of this course is to give you the basic tools to succeed in calculus, whether you're in high school, college or self-studying calculus!

This part of the course is divided into seven (7) "bite size" Parts. Here is what you'll learn about limits:

- **Day 1: Intuitive Idea Behind Limits and Notation**
- **Day 2: Basic Rules and Problems With Limits**
- **Day 3: Limits by Factorization**
- **Day 4: Limits by Rationalization**
- **Day 5: Squeeze Theorem**
- **Day 6: Trigonometric Limits**
- **Day 7: Limits at Infinity**

Thanks for signing up, and I hope you enjoy the course!

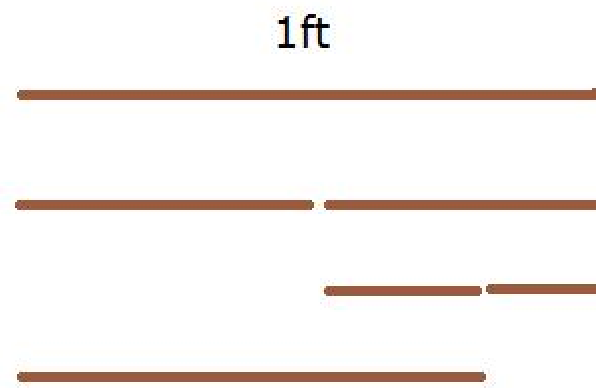
Now, get ready, we're starting now...

2 A First Intuition

The focus of this first lesson is the intuitive idea behind limits. We'll review the intuitive idea, or learn it for the first time if you never got it!

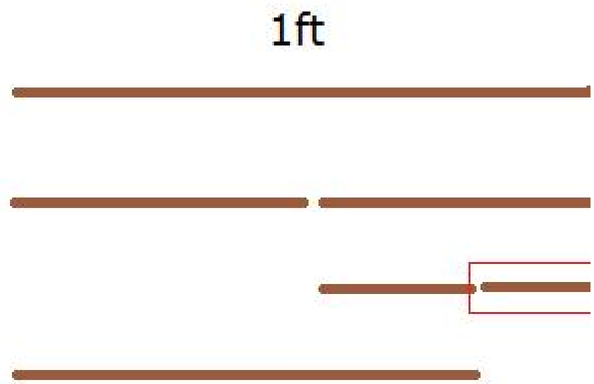
I always like to put examples in order to give an intuitive idea of limits. That is because we already have at least some intuition buried there in our mind of what we mean by limits.

So, let's do a little thought experiment. Suppose you have a stick, 1 feet long. Now, you cut it in half. You take one half, and cut it in half again. You take that half's half and glue it to the first half. A picture would help here:



The first one is the stick, 1 feet long. Then you have the stick taken appart in two halves. Then you have the second half cut in two halves. And the fourth image is one of these halves glued to the first half.

What if you continue this process? That is, you take the following half that remains:



And cut it in half? And then glue it to the other one, of course. What if you continue this indefinitelly? You'll approach the original 1 feet stick. If you stop at any point in this process, you won't have 1 feet, you'll have smaller number. But if you continue you'll always approach 1 feet more and more.

In this case we say that the limit is 1 feet.

And that's the intuitive idea of limits! The limit is the thing we are approaching, when something else is happening.

In terms of functions of one variable, we have an independent variable, usually denoted with the letter x. To specify the limit of a function, we tell what the function is approaching when the independent variable x approaches a constant (this is the something else happening).

In the example of the 1 feet stick, the independent variable would be the number of times we repeated the process of taking one half and cutting it.

So, the notation for expressing the limit of a function when the independent variable x approaches a constnt a is:

$$\lim_{x \to a} f(x) = L$$

This is read: The limit of $f(x)$ as x approaches a equals L.

This notation may seem confusing at first, but you'll quickly get used to it.

3 Why Limits Are Useful

You might ask what this is useful for. Very good question. Why would you need to know what the function is approaching? You can easily calculate the value of the function at the exact point, right?

Well, the point is that we don't care what the function is at that particular point x in many cases!

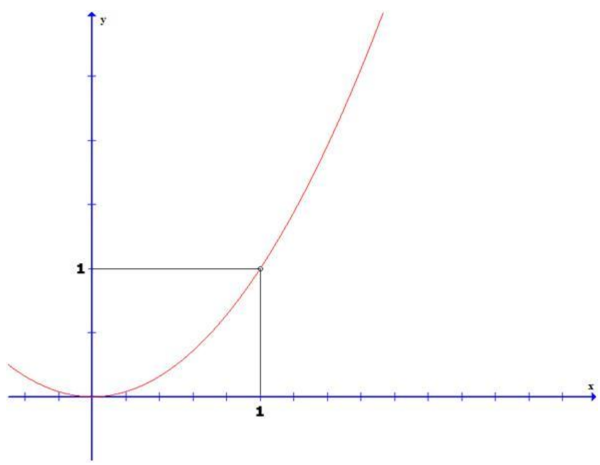
This is either because we don't really care, or because the function is not defined at that point x.

To show this, let's consider the following function:

$$f(x) = \begin{cases} x^2 & \text{if } x \neq 1 \\ 0 & \text{if } x = 1. \end{cases} \tag{3.1}$$

Don't let this notation intimadate you! This only means that this function equals x^2 when x is anything other than 1, and equals 0 when $x = 1$. To learn more about this type of functions and how to treat them you might be interested in the upgraded version of The Intuitive Calculus Course, where we learn in detail about these functions and solve many problems related to them.

This function looks like a parabolla, but has a hole at point $x = 1$. Here is its graph:



What does the function approach when x approaches 1? It also approaches 1, right? It doesn't matter that the function is other than 1 at that point! So,

$$\lim_{x \to 1} f(x) = 1$$

In calculus, the most useful limits are like this one. The value of the function at the specific point we care about is not defined, like 0/0 (which is complete junk), or useless, like zero or infinite.

In these cases we can know what the function is approaching, and that is what we really need.

Tomorrow we'll learn more about what limits are and how to solve simple problems.

4 Following Days

- **Day 2: Basic Rules and Problems With Limits**
- **Day 3: Limits by Factorization: Avoid These Common Mistakes**
- **Day 4: Limits by Rationalization: The Basic Trick For Solving Any Limit**
- **Day 5: Squeeze Theorem: Getting The Intuition and Why It Is Useful**
- **Day 6: Trigonometric Limits: These Can Be Tricky, Here Are All The Secrets**
- **Day 7: Limits at Infinity: Master The Basic Technique**