Intuitive-Calculus.com Presents

The Free Intuitive Calculus Course

Limits

Day 2: Basic Rules and Problems With Limits

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1 Welcome

Welcome to Day 2 of the Free Intuitive Calculus Course! The main purpose of this course is to give you the basic tools to succeed in calculus, whether you're in high school, college or self-studying calculus!

Today we focus on the basic rules governing limits and how to solve simple problems.

2 Why Limits Matter

The concept of limit of a function is the most important of all calculus. It is used to define derivation and integration, which are the main ideas of calculus.

As we said yesterday, the limit of a function is what the function approaches when the input (the variable x in most cases) approaches a specific value.

One of the questions in day 1 was: Is the limit of a function useful? Very good question. It is indeed useful. Many times we find functions that are undefined at certain values. This means that the function equals something like 0/0, or $\frac{\infty}{\infty}$ for specific values of x.

We don't know what those expressions mean!

Using limits, however, we can know what the function is approaching when the variable x approaches that value. We don't need to care whether or not the function is defined at that point.

You'll understand this better the more examples you see. That's why we now turn to...

3 Solving Simple Limits

Many limits are very easy to solve. Let's start with this one:

$$\lim_{x\to 1} 3x^2$$

Let's think. What will happen to the function when x approaches 1 more and more? Let's take out our calculator and make a table:

x	f(x)
0.90	2.4300
0.91	2.4843
0.92	2.5392
0.93	2.5947
0.94	2.6508
0.95	2.7075
0.96	2.7648
0.97	2.8227
0.98	2.8812
0.99	2.9403
1.00	3.0000

The function clearly approaches 3, right?

Let's see what happens if x approaches 1, but takes values greater than one:

x	f(x)
1.10	3.6300
1.09	3.5643
1.08	3.4992
1.07	3.4347
1.06	3.3708
1.05	3.3075
1.04	3.2448
1.03	3.1827
1.02	3.1212
1.01	3.0603
1.00	3.0000

It also approaches 3...

So, when this happens, we write:

$$\lim_{x\to 1} 3x^2 = 3$$

In this case it was easy to find the limit by simply calculating values of the function with x approaching one. We now turn to more practical methods. Remember, though, that you can always check your answers in this manner, if you have a calculator.

4 Limit of the Sum of Functions

Now, let's suppose we have two functions:

$$f(x) = x^2, \quad g(x) = 6$$

$g(x)$ does not depend on x, because it is constant. This means its value is six, no matter what the x is.

What will happen if we add these functions and try to find the limit as x approaches 2?

$$\lim_{x \to 2} [f(x) + g(x)] = \lim_{x \to 2} \left[x^2 + 6\right] = ?$$

We don't need to make a table to know that when x approaches 2, x^2 will approach 4. Six always will be six. So:

$$\lim_{x \to 2} \left[x^2 + 6\right] = 2^2 + 6 = 10$$

Here we can note two important properties of the limit, that hold for any function, not just these ones:

$$\lim_{x \to 2} \left[x^2 + 6\right] = \lim_{x \to 2} x^2 + \lim_{x \to 2} 6 = 2^2 + 6 = 10$$

The first one is that **the limit of the sum of two or more functions equals the sum of the limits of each function.**

The second one is that **the limit of a constant equals the same constant. By a constant we mean any fixed number.**

5 Limit of a Product

In our first example we had that:

$$\lim_{x \to 1} 3x^2 = 3$$

To solve this limit we can use another important property of the limit of a function. Can you see which one?

$$\lim_{x \to 1} 3x^2 = \lim_{x \to 1} 3 . \lim_{x \to 1} x^2 = 3.1 = 3$$

This is similar to the property about sums, but with products: **The limit of the product of two or more functions equals the product of the limits of each function.**

This also means that whenever you have a function multiplied by any number you can do this:

$$\lim_{x \to 1} 3x^2 = 3 \lim_{x \to 1} x^2$$

That is, you can take the number out of the limit sign. Another example:

$$\lim_{x \to 2} 5x^3 = 5 \lim_{x \to 2} x^3 = 5.2^3 = 40$$

6 Limit of a Quotient

As you may probably expect by now, the limit of the quotient of two functions equals the quotient of the limits. For example:

$$\lim_{x\to 1}\frac{6x+4}{3x-1}=\frac{\lim_{x\to 1}(6x+4)}{\lim_{x\to 1}(3x-1)}=$$

$$=\frac{6+4}{3-1}=5$$

In this example we used all the properties we learned. Using these you can solve many simple limits.

At first, you should think what properties you are using to solve your limits. But as you practice more, you'll see that you can simply replace x for the value it is approaching (in these simple exercises).

Let's see another example:

$$\lim_{x\to 0}\frac{3x^2+5}{x+1}=\frac{3.0^2+5}{0+1}=5$$

In this way you can solve but the simplest of limits. Tomorrow we'll start to learn elaborate techniques for solving more complex limits. For now, I leave you with the following problems.

7 Exercises

Solve the following limits:

1. $\lim_{x\to 0}(x-2)(x+3)$
2. $\lim_{x\to -2}\frac{2-x}{x+1}$
3. $\lim_{x\to 1}\frac{(x^2+2x+1)}{x-2}$
4. $\lim_{x\to \frac{1}{2}}\left[\frac{(x+\frac{3}{2})}{x-1}+\frac{1}{2}\right]$
5. $\lim_{x\to -1.5}\frac{x-1}{x+1}$

8 Following Days

- **Day 3: Limits by Factorization**
- **Day 4: Limits by Rationalization**
- **Day 5: Squeeze Theorem**
- **Day 6: Trigonometric Limits**
- **Day 7: Limits at Infinity**