Intuitive-Calculus.com Presents

The Free Intuitive Calculus Course

Limits

Day 3: Limits by Factorization

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1 Welcome

Welcome to Day 3 of the **Online Intuitive Calculus Course!** The main purpose of this course is to give you the basic tools to succeed in calculus, whether you're in high school, college or self-studying calculus!

Today we focus on one of the basic techniques for solving limits: **factorization.**

2 A First Example

One of the basic techniques for solving limits is to try to factor the numerator and denominator and see what is left. We must note that often we make assumptions about the function in order to factor and simplify. For example, we assume that the factor we are simplifying isn't zero. Let's see some examples.

$$\lim_{x\to 2} \frac{x^2-4}{x-2}$$

We see that for $x = 2$ our function is indeterminate (0/0). But we don't care about that because we are taking the limit, not evaluating the function. So, we always consider $x \neq 2$. We note that we can factor the numerator:

$$\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{(x-2)(x+2)}{x-2}$$

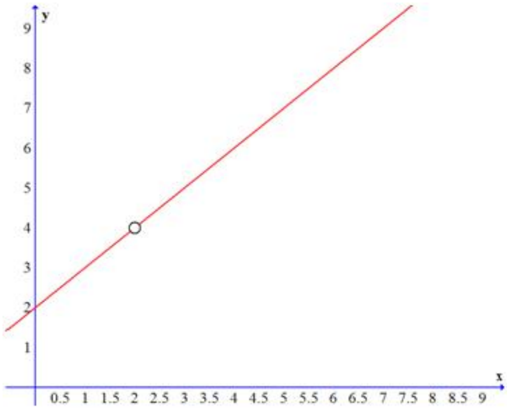
As $x \neq 2$, we have that the factor $(x-2) \neq 0$. Then we can simplify that expression to:

$$\lim_{x\to 2}(x+2)$$

And using the properties we learned yesterday:

$$\lim_{x\to 2}(x+2) = \lim_{x\to 2} x + \lim_{x\to 2} 2 = 2 + 2 = 4.$$

If we graph this function we get that it is, as a previous example, a line with a hole. In this case, the hole is at $x = 2$:



In every point except at $x = 2$, this function equals the line $y = x + 2$.

3 The Secret For Using This Technique

The secret for using this technique is identifying the limits to which you can apply it! The algebra is easy, since you must be already a master of algebra...

But here's the real secret:

Whenever you see the limit of the quotient of two polynomials, the first thing you must look is at what value the limit is taken (what value x approaches, let's call it a). It is often the case that the numerator and/or denominator becomes zero at that point. If that is the case, the factor $(x - a)$ *should appear there.*

If you remember these things, you might recall that this is the consequence of the **Factorization Theorem in Algebra**. Never mind, let's do another example:

4 A Second Example

$$\lim_{x\to 1}\frac{x^3-1}{x-1}$$

Here again, if we plug 1 into the function we'll get 0/0. But we can factor here:

$$\lim_{x\to 1}\frac{x^3-1}{x-1}=\lim_{x\to 1}\frac{(x-1)(x^2+x+1)}{x-1}$$

Here we can simplify the $(x-1)$ factors because $x\neq 1$:

$$\lim_{x\to 1}\frac{(x-1)(x^2+x+1)}{x-1}=\lim_{x\to 1}(x^2+x+1)=1^2+1+1=3.$$

Now I think you are ready to solve the following problems...

5 Exercises

Solve the following limits by factorization:

1. $\lim_{x\to 3} \frac{x^2+x-6}{x-3}$
2. $\lim_{x\to 1} \frac{x^2-2x+1}{x-1}$
3. $\lim_{x\to 2} \frac{x^2-5x+6}{x^2-12x+20}$
4. $\lim_{x\to 1} \frac{x^n-1}{x-1}$

6 Still To Come

- **Day 4: Limits by Rationalizatio**
- **Day 5: The Squeeze Theorem**
- **Day 6: Trigonometric Limits**
- **Day 7: Limits at Infinity**