Intuitive-Calculus.com Presents

The Free Intuitive Calculus Course

Derivatives

Day 8: Geometric Idea of The Derivative

By Pablo Antuna

Contents

1 Welcome

Welcome to **Day 8** of the **Intuitive Online Calculus Course!** The main purpose of this course is to give you the basic tools to succeed in calculus, whether you're in high school, college or self-studying calculus!

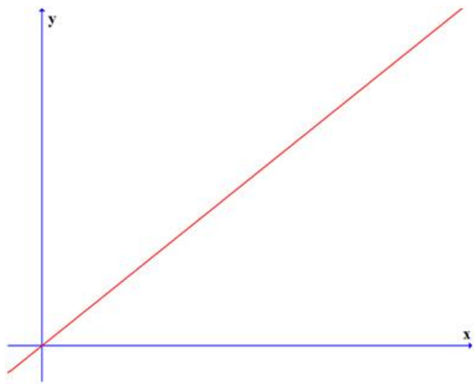
Today we begin to study derivatives. This part of the course is divided into the following days:

- **Day 8: Geometric Idea of The Derivative**
- **Day 9: Physical Interpretation of the Derivative**
- **Day 10: Finding Derivatives by Applying the Definition**
- **Day 11: Derivatives of Trigonometric Functions**
- **Day 12: The Chain Rule**
- **Day 13: The Product Rule and Quotient Rule**
- **Day 14: Implicit Differentiation and Inverse Trigonometric Functions**

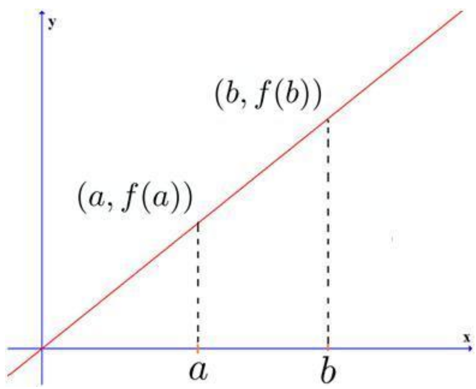
2 A Little Review

The derivative may seem a very abstract concept at first. However, it is the generalization of a simple concept: **slope**. We first learn about slope in algebra, and the concept of the derivative is just a slight generalization.

We will first of all review the concept of slope as you see it in algebra. Let's consider the graph of a straight line:

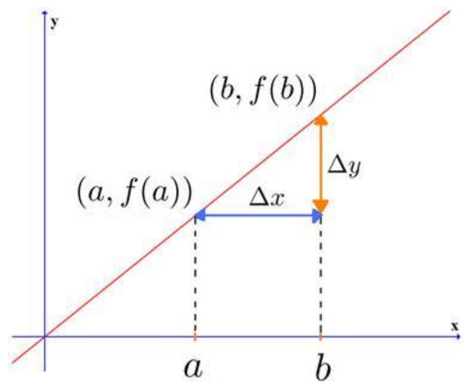


To find the slope of this line we proceed as follows: Choose two points on the x axis, and take the corresponding coordinates in the line:



The slope of this line is simply defined as the quotient between the variations of the y coordinate and the x coordinate. That is:

$$m = \frac{\Delta y}{\Delta x}$$



Here the letter m denotes the slope. Looking at the graph we can find a more explicit expression for both Δy and Δx. Δy would be the differences between the y coordinates of the two points on the line:

$$\Delta y = f(b) - f(a)$$

And Δx would be the differnece between the two x coordinates:

$$\Delta x = b - a$$

So, the expression for our slope becomes:

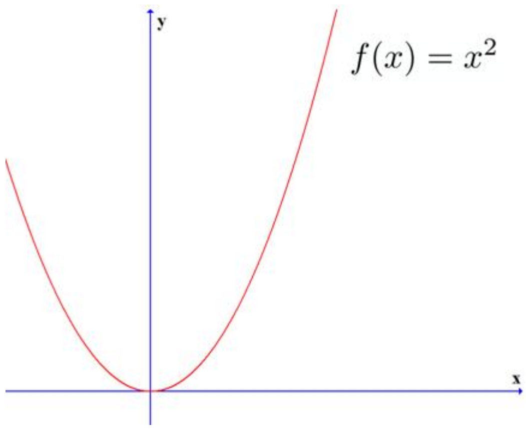
$$m = \frac{f(b) - f(a)}{b - a}$$

This is an expression that would become very familiar to you as you learn more about derivatives.

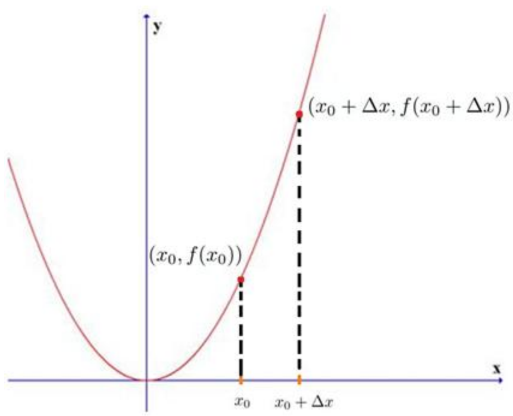
Ok, that is all we need to remember about algebra, now we turn to a more interesting problem...

3 Finding the Slope of A Curve

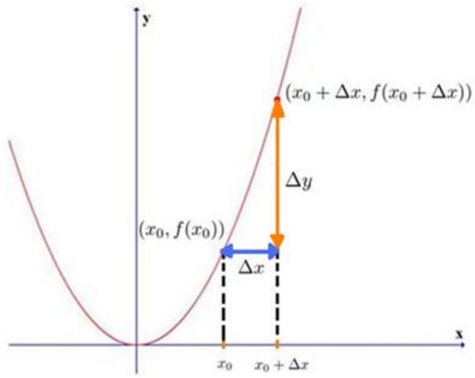
A more interesting (and difficult) problem is to try to define and find the slope of a more general curve, not necessarily a line. As an example, we'll consider the graph of the function $f(x) = x^2$:



We can proceed in a similar manner and choose two points on the x axis. But this time, we'll choose a *fixed* point x_0 and a *moving point* $x_0 + \Delta x$. This Δx can be any number, positive or negative:



And there we have the *change* of both the y coordinate and x coordinate:



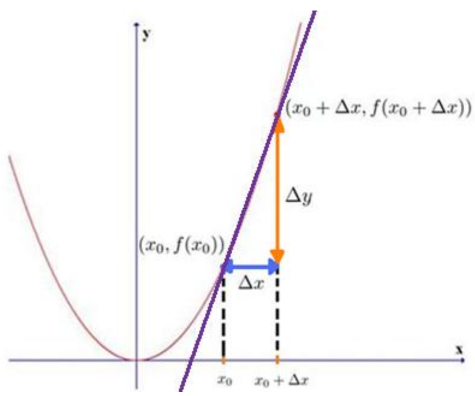
We always use the letter Δ (delta) to denote the change in a variable. We can express Δy as:

$$\Delta y = f(x_0 + \Delta x) - f(x_0)$$

If we take the quotient between the change in y and the change in x, we'll have the slope of the *secant line*:

$$m = \frac{\Delta y}{\Delta x} = \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

This line is called a secant line because it touches the curve in two points:



However, we're not interested in finding the slope of the secant line, but the slope of the curve. So, what we do in this case, is to move the point $x_0 + \Delta x$ more and more to the left (in this case). In this process, the secant line will become a tangent line. See an animation of this.

What we are doing is making $\Delta x \to 0$. That is, we take the limit of the slope of the secant line as Δx approaches zero. And this is how we define the derivative at point x_0:

$$f'(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

This is read: *f prime of x_0*. What we obtain by taking this limit is the slope of the tangent line. And this is how we define the *slope of a curve*.

In reality, curves don't have a slope, but we can take the slope of the tangent line. We call this slope the **derivative of the function at x_0.**

One important difference with the previous case in which we considered a straight line is that the derivative can be different at each point x_0. That is, we get a function f', called the derivative of the function f, which depends on the point x_0.

In the case of a straight line, we the same value at every point. In the case of a more general function that may not be the case.

What I want you to leave with is the geometrical interpretation of the derivative: **the slope of the tangent line.**

Tomorrow, we'll start to delve into the physical interpretation of this concept.

4 Still To Come

- **Day 2: Physical Interpretation of the Derivative**
- **Day 3: Finding Derivatives by Applying the Definition**
- **Day 4: Derivatives of Trigonometric Functions**
- **Day 5: The Chain Rule**
- **Day 6: The Product Rule and Quotient Rule**
- **Day 7: Implicit Differentiation and Inverse Trigonometric Functions**